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On Application of the Gompertz Relational Model to Estimate Fertility from the **CEB** Data of 1981 Census of India

1. Introduction

IN absence of the adequate and complete data from the civil registration system, the census age data appear to be the viable source for indirectly estimating the level of fertility in the population during the intercensal period. The 1981 Census of India, however, included a question, to all ever married women on the children ever born (CEB) and to all currently married women on the births during the last one year. One can, therefore, utilize these information to estimate the level and age pattern of fertility. An attempt is made here to utilize the CEB data to estimate the age-specific fertility rates using a relational Gompertz Model (RGM) following Zaba's procedure (Zaba, 1981) which is a modification over the one given by Brass (1979). The validity of the estimates so obtained arc checked by comparing them with the ASFRs of 1972 and 1978 surveys (R.G.I 1977, 1981). In addition, some methodological problems involved in the application of this procedure and their impact on the estimates are also discussed.

2. Description of Method

Recently, Zaba (1981) developed a procedure known as "Ratio Method" to estimate the age-specific fertility rates from the average parities (P_i,) by age of women. If the pattern of fertility of any population follows a Gompertz curve, the cumulated fertility upto certain age *x[F(x)]* is expressed as

$$F(x) = T\,e^{-e^{-[\alpha+\beta y_s(x)]}} \qquad (1)$$

where T = total fertility rate

$$Y_s(x) = -\text{Log}_e\left[-\text{Log}_e\left(\frac{F_s(x)}{F_s}\right)\right]$$

$F_s(x)$ and F_s are the cumulative fertility upto age x and total fertility rate in the standard population respectively. α and β are the parameters of the model.

It may be observed from the equation (1) that one needs estimate of T to estimate the ASFRs, which is generally not available. To overcome this problem Zaba defines the following ratio

$$Y_x = -\text{Log}_e\left[-\text{Log}_e\left(\frac{F(x)}{F(x+5)}\right)\right]$$

In this expression, average parity by age can also be used in place of the cumulated fertility so that

$$Y_i = -\text{Log}_e\left[-\text{Log}\left(\frac{P_i}{P_i+1}\right)\right] \tag{2}$$

where P_i is the average parity in the ith age group. By simple mathematical manipulation, after substituting P_i's in the above equation and expanding the function by Taylor's series we have

$$Y_i + \phi_i'(1) - \phi_i(1) = \alpha + (\beta - 1)^2 \cdot \frac{C}{2} + \beta\,\phi_i'(1) \tag{3}$$

where $\phi_i(1)$ and $\phi_i'(1)$ [first derivative of $\phi_i(1)$] are given by Zaba (1981). C is the second derivative of $\phi_i(1)$ which is constant with respect to age especially from 20 to 40 years of age. It can be seen from this equation that the plot of points $Y_i + \phi_i'(1) - \phi_i(1)$ (called P points) against $\phi_i'(1)$ would give a straight line. When β is very close to unity, the second term in equation (2) would be zero or negligible. In other words, the constant term in the straight line would equal α. In fitting equation (2), however, only those points are considered which lie on the straight line. With the help of estimated values of α and β one can estimate T and age-specific fertility rates as follows :

$$T_i = P_i/\hat{P}_i \qquad i = 1, 2, 3$$

where

$$\hat{P}_i = \bar{e}^{\bar{e}[\alpha+\beta z_s(x)]}$$

$Z_s(i)$ denotes standard value corresponding to P_i. The estimate of T may be taken as an average of $\hat{T}_i$ (i : 1, 2, 3 . . . 7) is a suitable age range, say 20 to 35. Thus the cumulated fertility upto age x can be obtained as

$$F(x) = \hat{T}\,\bar{e}^{\bar{e}\,[\alpha+\beta y_s(x)]} \tag{4}$$

From the cumulated fertility rates, age-specific fertility rates can easily be obtained by using the successive subtraction and division of the result by 5.

3. Analysis and Discussions

The estimates of TFR for 1972 and 1978 surveys are used to study the trends in TFR and thereby to check the consistency of the estimates for 1981 obtained by using Zaba's Method. No attempt, however, is made to study the age pattern of fertility. The estimates of TFR for 1972 Fertility Survey of India are obtained by using the age-specific marital fertility rates and the marital distribution from 1971 census. This step is carried out just to adjust the rates for the differences in the marital distributions in the survey and the 1971 census. We feel that this ensures the better comparability among these rates and the one obtained from the 1981 census data.

The estimates of TFR from the CEB data from 1981 census come out to be 4.93, 4.45 and 4.81 respectively for the rural, urban and the total population.

TABLE 1—THE ESTIMATE OF TOTAL FERTILITY RATE FROM CHILDREN EVER BORN USING RELATIONAL GOMPERTZ MODEL, INDIA AND STATES, 1981

Country/State	*Estimated Value of TFR*		
	Rural	*Urban*	*Combined*
India	4.93	4.45	4.81
Andhra Pradesh	4.16	3.89	4.13
Bihar	4.44	4.29	4.42
Gujarat	5.26	4.50	5.04
Haryana	6.33	5.04	6.12
Karnataka	4.93	4.47	4.85
Kerala	4.82	4.08	4.60
Madhya Pradesh	5.49	5.00	5.40
Maharashtra	5.06	4.18	4.79
Orissa	4.68	4.55	4.66
Punjab	5.11	4.35	4.90
Rajasthan	5.89	5.32	5.79
Tamil Nadu	4.18	4.12	4.18
Uttar Pradesh	5.18	4.91	5.14
West Bengal	5.37	4.07	5.03

These estimates of TFR are considerably higher than the ones obtained directly from the current fertility data (births during last one year) of 1981 census which are 3.9, 2.8 and 3.6 respectively. Undoubtedly, the current fertility data of 1981 census grossly underestimates the fertility level as compared to SRS results of 1978 (4.8, 3.4 and 4 5 respectively). During 1978-81, one may not expect fertility decline of this magnitude due to the set-back which family planning programme suffered during the post emergency period. Compared to the indirect estimates, extent of under-reporting in current fertility data of 1981 may be estimated to be of order 26, 59 and 34 per cents respectively for the rural, urban and the total population. This also suggests that the under-reporting is more in the urban areas. But it may be observed that for the rural area, indirect estimate (4.93) is quite close to that from the SRS (4.80) whereas for the urban area, there is a difference of almost 1.1. This indicates the probability that Zaba's method over-estimates the TFR for urban areas.

The analysis for the states, especially where fertility has declined faster, indicates even greater ambiguity in estimation of fertility. Table 2 gives the

TABLE 2—THE ESTIMATES OF TOTAL FERTILITY RATE FROM AGE-SPECIFIC MARITAL FERTILITY RATES, INDIA AND STATES, 1972

Country/State	*Rural*	*Urban*	*Combined*
India	5.99	4.80	5.74
Andhra Pradesh	4.91	4.05	4.72
Bihar	5.27	4.22	5.17
Gujarat	6.80	4.96	6.26
Haryana	7.61	5.25	7.16
Karnataka	4.89	3.66	4.57
Kerala	4.74	4.23	4.66
Madhya Pradesh	7.23	5.88	6.72
Maharashtra	5.25	4.30	4.95
Orissa	5.10	4.43	5.05
Punjab	6.06	4.94	5.77
Rajasthan	7.25	5.54	6.96
Tamil Nadu	4.55	3.68	4.28
Uttar Pradesh	7.02	5.78	6.89
West Bengal	—	3.91	—

TFR for 1972. This is obtained by using the age-specific marital fertility rates and marital distribution of 1971 census. It may be worthwhile to mention that like all-India, fertility levels from direct question on births during last one year in 1981 census have been made available so far only for Haryana, Karnataka, Punjab, Rajasthan and Uttar Pradesh. As observed for India, the fertility level obtained directly for these states (Haryana, 1981, Karnataka, 1981, Punjab, 1981, Rajasthan, 1981, Uttar Pradesh, 1981) are significantly lower than the estimates presented in Table 1. The estimates in Table 1 are higher by as much as 39 per cent for Haryana; 29 per cent for rural Haryana, and 68 per cent for urban Haryana. For Karnataka, corresponding figures are respectively 73, 59 and 103 per cent higher. The extent of under-reporting for Punjab comes out to be 44, 42 and 61 per cent respectively for total, rural and urban areas. Nevertheless, these estimates are comparatively lower for Rajasthan (7, 2 and 36 per cent) and Uttar Pradesh (17, 13 and 36 per cent). It indirectly suggests that Zaba's method probably gives higher estimates; more so in the areas which have experienced faster fertility decline. This will be even more clear when we discuss below the trends in TFR over 1972-81 and 1972-78.

TABLE 3—PERCENTAGE DECLINE IN TOTAL FERTILITY RATES DURING 1972-81, INDIA AND STATES

Country/State	*Rural*	*Urban*	*Combined*
India	17.70	7.29	16.20
Andhra Pradesh	15.27	3.95	12.50
Bihar	15.75	−1.66	14.51
Gujarat	22.65	9.27	19.49
Haryana	16.82	4.00	14.53
Karnataka	−0.82	−22.13	−6.12
Kerala	−1.69	3.55	1.29
Madhya Pradesh	24.07	14.97	19.64
Maharashtra	3.62	2.79	3.23
Orissa	9.02	−2.71	7.22
Punjab	15.68	11.94	15.77
Rajasthan	18.75	3.97	16.81
Tamil Nadu	8.13	−11.96	2.34
Uttar Pradesh	26.21	15.03	25.37
West Bengal	—	−4.09	—

Kerala, known to have experienced lower fertility, comes at 4th position after Andhra Pradesh, Bihar and Tamil Nadu. If one agrees with the hypothesis that the higher age at marriage, higher literacy, higher percentage of couples protected mean lower fertility, the estimates of TFR for Kerala and in the urban areas of some states are certainly not acceptable. Two points need to be emphasised. First the data on CEB is contaminated to such an extent even in younger age groups for the states like Andhra Pradesh, Bihar and Uttar Pradesh that Zaba's method under-estimates the fertility for such states. Secondly the estimates of TFR may come out to be on higher side for those states where age pattern of fertility has undergone substantial change over a period of time. Kerala may be an example.

The trends in fertility over 1972-81 and 1972-78 may be visualized from Table 3 and 4 respectively. The percentage decline in the TFR over the period of 1972-81 for India comes out to be 18 per cent for the rural, 7 per cent for the urban and 16 per cent for the total population. It may also be observed that fertility decline for all the states as well as for all India, is invariably

TABLE 4—PERCENTAGE DECLINE IN TOTAL FERTILITY RATE DURING 1972-78, INDIA, AND STATES

Country/State	*Rural*	*Urban*	*Combined*
India	19.87	29.17	21.60
Andhra Pradesh	6.30	18.52	6.78
Bihar	—	—	—
Gujarat	23.53	25.40	23.32
Haryana	32.98	37.14	34.36
Karnataka	16.16	34.43	16.85
Kerala	36.71	36.17	36.23
Madhya Pradesh	22.54	37.07	21.13
Maharashtra	29.52	27.91	29.29
Orissa	11.76	16.48	10.89
Punjab	30.69	33.20	30.68
Rajasthan	26.90	29.60	26.72
Tamil Nadu	16.48	18.48	18.22
Uttar Pradesh	11.68	25.61	12.90
West Bengal	—	—	—

higher for the rural area than those observed for the urban area. For some states, indirect estimates of TFR for 1981 seem to be higher than the TFR in 1972. It is interesting to note that the states where fertility has declined minimally (Table 3) have faired quite well, not only in the family planning programmes but also in other developmental programmes. Two points clearly emerge from this. First, the estimates of TFR for these states and for the urban areas in 1972 are grossly under-estimated i.e. quality of data is very poor. This may not be accepted because the reporting of events are related to the level of socio-economic development. Secondly, as mentioned earlier, the TFR for such states where fertility declined faster during 1970s, is over-estimated.

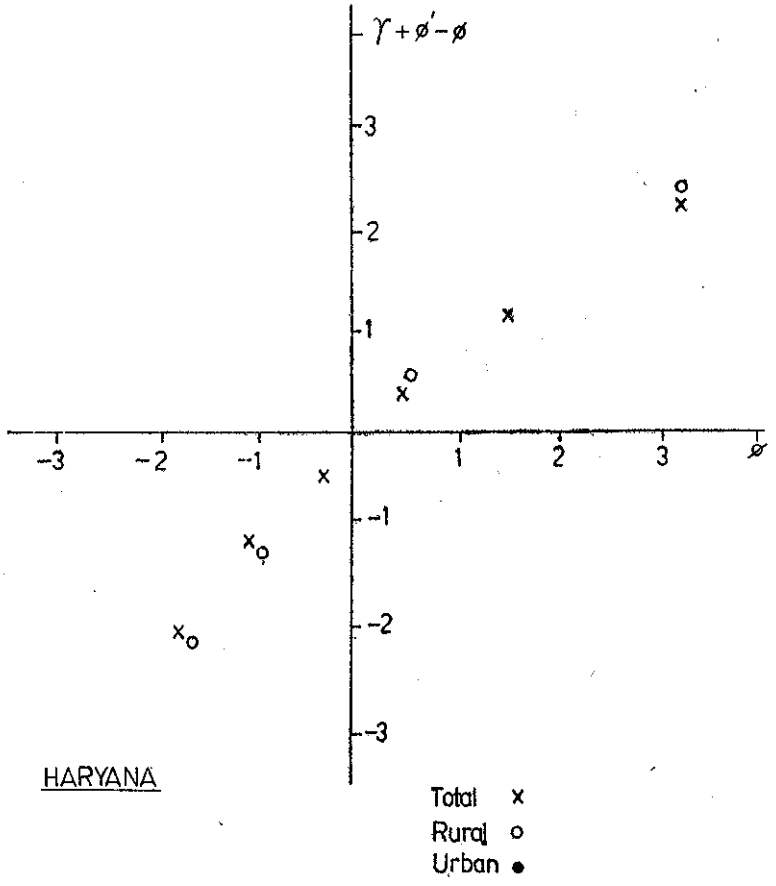
Unlike Table 3, the Table 4 reflects completely different picture. We expect the pattern of decline by states, rural, and urban to be the same both during 1972-81 and 1972-78. All the states which seem to have experienced slower fertility decline (Table 3) show substantially faster decline (Table 4). On the other hand, in urban areas, the fertility decline is invariably higher than that observed for the rural area. Why does this type of anomaly arise between Tables 3 and 4. Whether this is an artifact of the methodology or of data directly obtained from the survey or the census. Some of these problems are discussed in the next section.

4. Methodological Problems

To understand the theoretical as well as the practical problems in the application of Zaba's method to Indian situation we present three graphs respectively for Haryana, Gujarat and Tamil Nadu. We have not given here the graphs for other states because we encounter almost the similar problems. For these graphs, P-points $[(\gamma_i + \phi_i'(1) - \phi_i(1)]$ are plotted against $\phi_i'(1)$. In general, however, only those points are selected for estimating the parameters α and β which lie on a straight line. Simple least squares method is also used to fit a line on the basis of some selected points.

The selection of points, however, seems to be partially arbitrary. It may be observed from these figures that there are more than one combinations of (P, ϕ) points which would form a straight line. Each combination would provide different estimates of α and β, thereby giving not only the different levels of fertility but also the different age patterns of fertility. Mention may also be made that we would have only two or three observations for fitting the line, because of the presence of unknown degree of the recall lapse in the data on CEB.

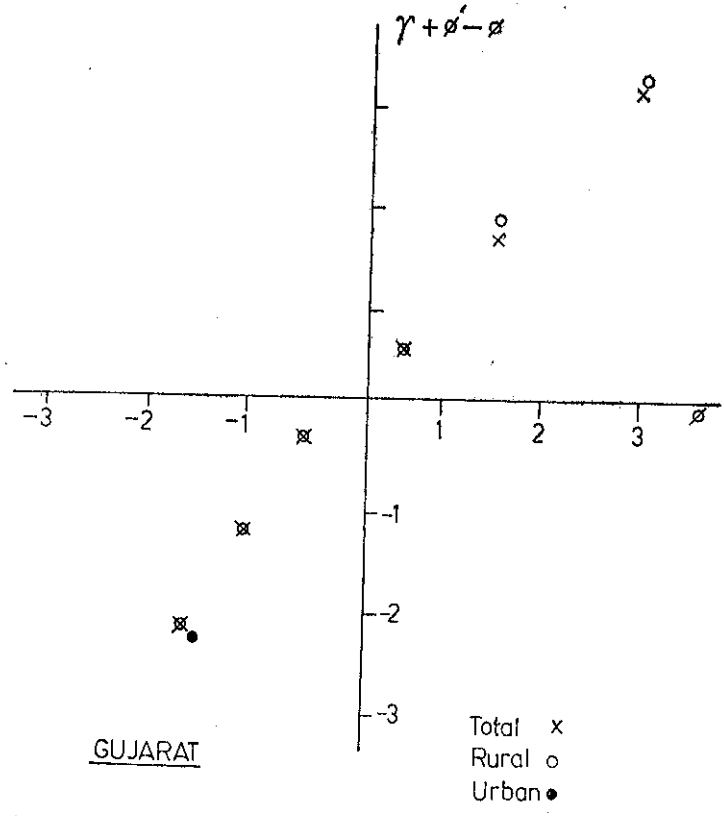
It has been observed (for Haryana, Karnataka, Kerala, Tamil Nadu) that the estimated level of fertility from some combinations comes out to be lower for the rural area than that for the urban area. It is also observed for some states that the TFR for the total area does not fall between TFR obtained for



Note : Some points relating to rural and urban population coincide with the corresponding points pertaining to the total population.

the rural and urban areas separately. The estimates provided in Table 1 for such states have been found after several trial and errors. Thus, the method sometimes may provide an arbitrary level of fertility and one has to be aware of the demographic characteristics (especially fertility levels) of the population in question in order to decide about the level of fertility and to judge its propriety—often a difficult task in wake of data of doubtful quality.

It has been pointed out that the fertility need not have been constant in the recent past. But it seems that the age pattern of fertility which yielded the reported pattern of average CEB and the pattern of current fertility is assumed to be same in use of Zaba's method. In other words, the current age pattern of fertility has remained constant in the past. This implicit assumption is questionable. This is widely known that during demographic transition, which



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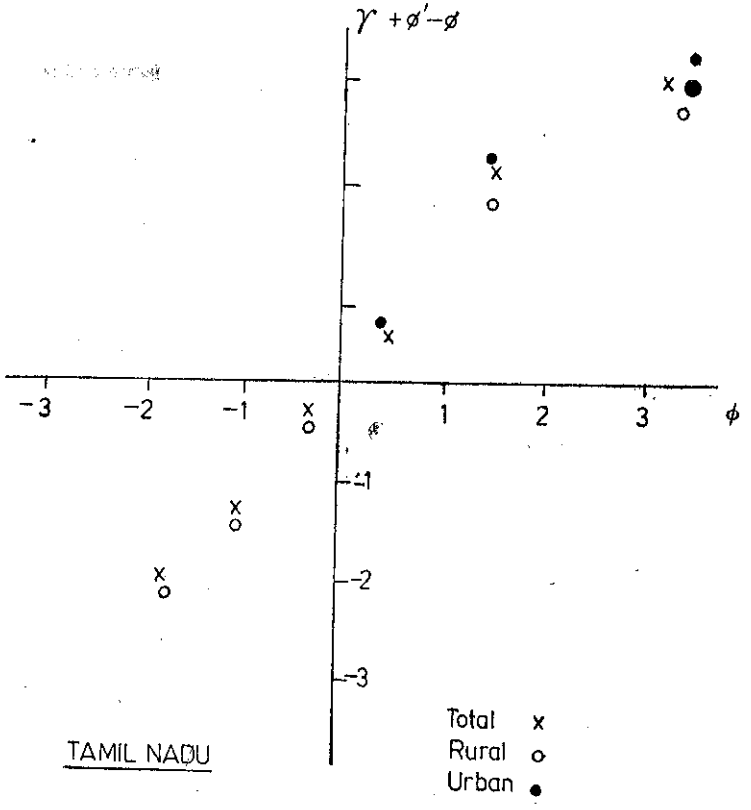
is taking place rapidly in almost all the developing countries, the age patterns of fertility undergo substantial changes (U.N. 1963, Coale and Trussell 1974). As a result of the transition, childbearing tends to be concentrated in a narrower range of the ages; say 20-35 years of age. So, the patterns of standard P_i's, $\phi_i(1)$ and $\phi_i'(1)$ may not reflect the patterns in the actual population. Probably due to this, the Zaba Method generally over-estimates the fertility when fertility is declining; Kerala, Tamil Nadu and some other states, especially the urban areas provide good examples to support this hypothesis.

5. Concluding Remarks

This paper attempts to estimate age-specific fertility rates from CEB data from 1981 census. The estimates of TFR obtained come out to be 4.93, 4.45 and 4.81 respectively for the rural, urban and the total population. It is observed that the extent of under-reporting of the births during the last one year



Note : Some points relating to rural and urban population coincide with the corresponding points pertaining to the total population.

reported in 1981 census seems to be around 26, 59 and 34 per cent for the rural, urban and the total population respectively. Under-reporting is more in the urban areas, which is probably an artifact rather than reality because the Zaba method over-estimates TFR for the areas where fertility in declining.

The study of trends with the estimates obtained by using Zaba's method may be misleading. The percentage decline in the TFR over the period of 1972-81 with such estimates comes out to be 18 percent for the rural area, and 7 percent for the urban area and 16 percent for all India. Even for the different states, fertility decline so estimated is invariably higher for the rural area than that observed for the urban area. It is observed that the states, where fertility decline is shown to be quite slow, have faired well not only in the acceptance of the family planning programme but also in achieving the targets of other developmental programmes.

The conclusion is that the estimates obtained from the application of Zaba's

method especially for the areas where fertility declined rapidly, should be interpreted with great care and in the light of ancillary data. In fact, on account of ratios utilised for estimating T, the risk of error in estimation becomes all the more grave. Further, Zaba's procedure might be viewed more as a smoothing procedure for the errors of age misreporting in the ASFRs than as a method of estimating the level of fertility. To what extent this smoothing is desirable depends again on the stage of demographic transition in the population.

It is also found that data on the births during the last one year in 1981 census of India happen to give a low estimate of fertility and the RGM gives the higher estimate of TFR in the population where fertility has declined. As such one is left with a dilemma as to whether the data on CEB and births during the last one year can be of any use in estimating fertility level in India. There seems to be no definite answer to these questions. Probably methods of estimating fertility rates by use of the two census age data or data on the CEB from the two consecutive censuses may fare better over these methods. Own children method may also be used which is more informative. However, none of these methods gives estimates for the current period. They provide the estimates on the average for the intercensal periods or a part thereof.

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